Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5 x 3 is the same as 3 x 5
- associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

YR

Early Learning Goal:
Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.
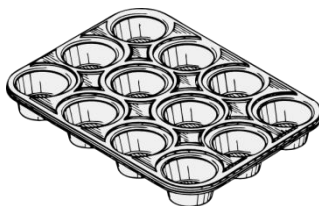
They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

Y1

End of Year Objective:
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

End of Year Objective:
Calculate mathematical statements for multiplication *(using repeated addition)* **and write them using the multiplication (x) and equals (=) signs.**

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:
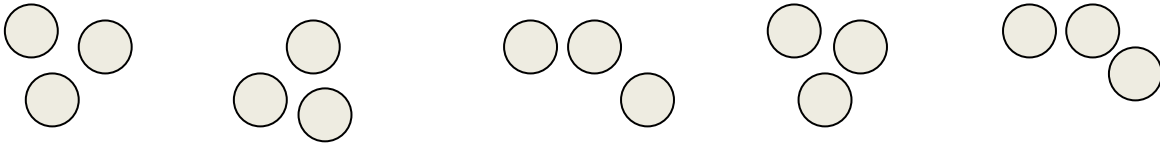
or in a more ordered pattern, with the groups of three indicated by the border outline:
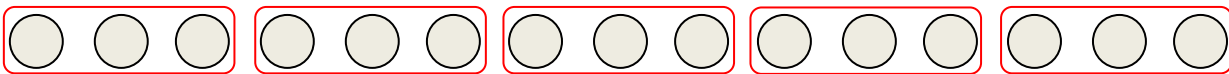
Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

5 x 3* can be represented as an array in two forms (as it has commutativity):
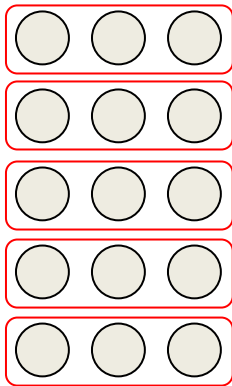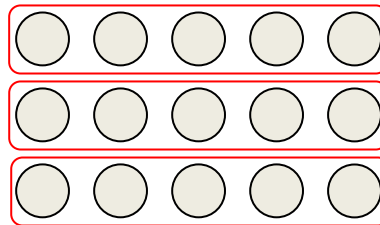
5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

*For mathematical accuracy 5 x 3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

End of Year Objective:
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

3 x 8

They may show this using practical equipment:
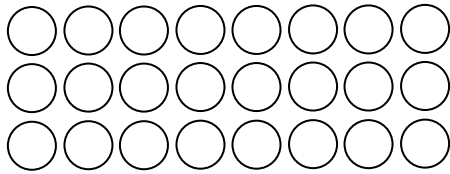
3 x 8 = 8 + 8 + 8 = 24

or by jottings using squared paper:

3 x 8 = 8 + 8 + 8 = 24

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and ones to help them. For example, when calculating 14 x 6, children should set out the array, then partition the array so that one array has ten columns and the other four.

10   4
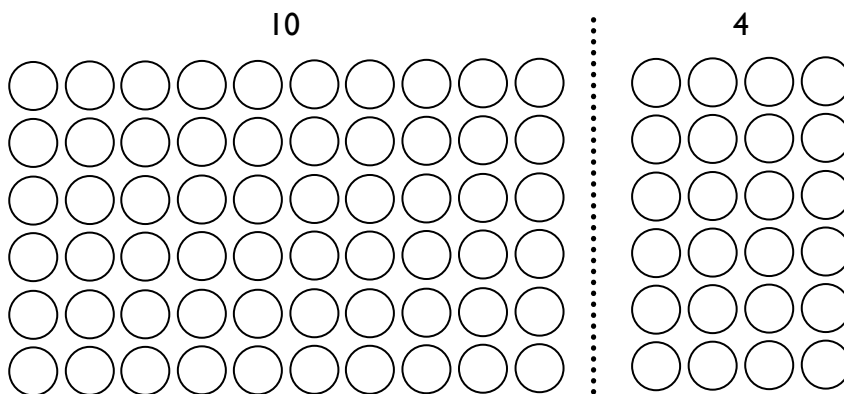
Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

(6 x 10) + (6 x 4)

= 60   +   24
= 84

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.
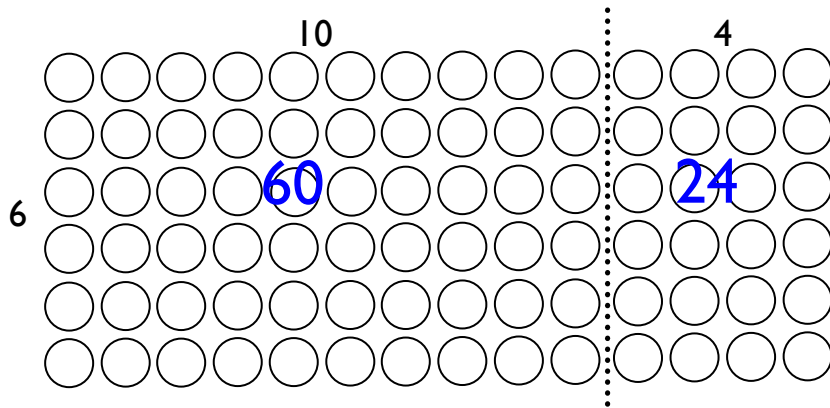
By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

x	10	4
6	60	24
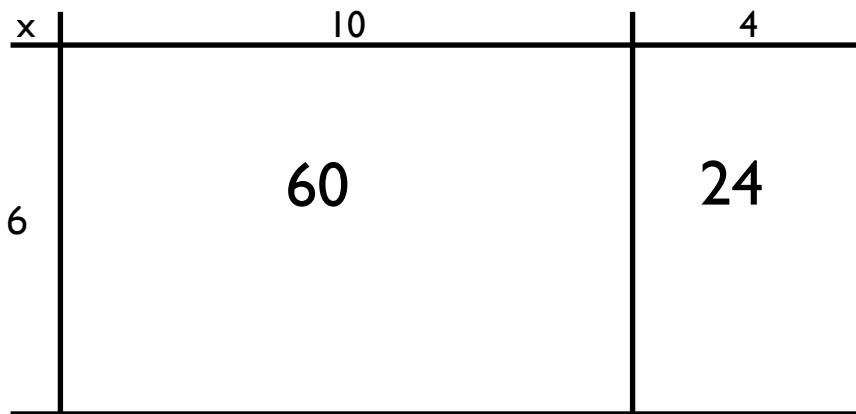
It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8

x	10	3
8	80	24

$$\begin{array}{r} 80 \\ +24 \\ \hline 104 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

$$\begin{array}{r} 180 \\ +42 \\ \hline 222 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y4

End of Year Objective:
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will either continue to use the grid method in Year 4, or will move forward to a formal written method of short division.

79×8

×	70	9
8	560	72

$$
\begin{array}{r}
560 \\
+ \quad 72 \\
\hline
632
\end{array}
$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

79×8 Using short multiplication

$$
\begin{array}{r}
79 \\
\times \quad 8 \\
\hline
72 \\
+ \quad 560 \\
\hline
632 \\
\hline
\text{\small 1}
\end{array}
$$

$$
\begin{array}{r}
79 \\
\times \quad 8 \\
\hline
632 \\
\text{\small 7}
\end{array}
$$

To use when children are more confident.

Remind children to always multiply the smallest value first. For year 4, this will be ones.

When using short multiplication, children must be aware of place value. To link to the grid method, children can partition and multiply eg: $8 \times 9 = 72$, $8 \times 70 = 560$ and then add the answers together using the column method.
Once children are confident in this method, they can multiply without partitioning.
Children must be aware of the value of the carrying number and ensure that it is written in the correct column.

By the end of the year, they will extend their use of the short multiplication method to be able to multiply three-digit numbers by a single digit number, e.g.

346×8

$$
\begin{array}{r}
346 \\
\times \quad 8 \\
\hline
2768 \\
\text{\small 3 4}
\end{array}
$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y5

End of Year Objective:
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the short multiplication method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

4346 x 8

```
      4346
  x      8
    34768
     2 3 4
```

Children must be aware of the value of the carrying number and ensure that it is written in the correct column.

Children should use long multiplication to multiply numbers with up to four digits by a two-digit number, e.g.

2693 x 24

```
      2693
  x      24
     10772
  +  53860
     64632
       1 1
```

When using long division, children must be reminded that we multiply the T and O separately. When we multiply the T, we must make our answer ten times bigger, so we put a 0 in the Ones column.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y6

End of Year Objective:
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
Multiply one-digit numbers with up to two decimal places by whole numbers.

By the end of Y6, children should be able to use the long or short multiplication method to multiply any number by a two-digit number. They should also develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

See Y5 for an example of long multiplication to multiply 4 digit numbers by 2 digit numbers.

12 X 1.25

```
      12.0000
   x   1.2500
       0.6000
       2.4000
    +12.0000
      15.0000
```

When multiplying decimal numbers by whole numbers, children must count the number of digits after the decimal point to understand where their answer should be written.
For example, 0.05 x 0.01 = 0.0005
Children must ensure that they are writing their answer in the ten thousandth column.

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.